Calmness of primal-dual solution maps in parametric Nash equilibrium

Matthieu Maréchal

We have considered a parametric Generalized Nash Equilibrium Problem, where each player $i \in \{1, \cdots, p\}$ solves the following optimization problem

$$\min_{x^i \in X_i(x^{-i})} \theta_i(x^i, x^{-i}, u), \tag{1}$$

where x^i is the decision variable of player i and u a parameter. The vector x^{-i} contains the decisions of all the players exept player i. The set $X_i(x^{-i})$ is the feasible set of decisions for player i, which depends on the decisions taken by the other players.

Define the set-valued map $S(u)$ as the set of solution of problem (1) with parameter u. The talk deals with the calmness of the set-valued map S at $(\bar{u}, \bar{x}) \in \mathrm{Gr}(S)$, where $\bar{u}$ is a fix parameter and $\bar{x}$ a solution of (1) with parameter $\bar{u}$. We say that S is calm at $(\bar{u}, \bar{x})$ if there exist constants $r, \varepsilon, L > 0$ such that, for all $u \in B(\bar{u}, \varepsilon)$ and for all $x \in B(\bar{x}, r)$, one has

$$\mathrm{dist}(x, S(\bar{u})) \leq L\|u - \bar{u}\|.$$

The approach consists in formulating the primal-dual problem into a Generalized Equation on the form

$$0 \in \Phi(x, \lambda, u)$$

where λ is the dual variable, and to study some sufficient conditions which ensure that this generalized equation is calm. The payoff functions of each agent are not supposed to be differentiable, this situation can be found in the electricity market. The techniques that we have used are based on the coderivatives introduced by Mordukhovich.

The talk proposes to apply this work to the electricity market, in order to estimate the influence of the thermal losses on the market.