

# On the Asymptotic Properties of Piecewise Contracting Maps

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## Abstract:

We study the asymptotic dynamics of maps which are piecewise contracting on a compact space. These maps are Lipschitz continuous, with Lipschitz constant smaller than one, when restricted to any piece of a finite and dense union of disjoint open pieces. We focus on the topological and the dynamical properties of the (global) attractor of the orbits that remain in this union. As a starting point, we show that the attractor consists of a finite set of periodic points when it does not intersect the boundary of a contraction piece, which complements similar results proved for more specific classes of piecewise contracting maps. Then, we explore the case where the attractor intersects these boundaries by providing examples that show the rich phenomenology of these systems. Due to the discontinuities, the asymptotic behaviour is not always properly represented by the dynamics in the attractor. Hence, we introduce generalized orbits to describe the asymptotic dynamics and its recurrence and transitivity properties. Our examples include transitive and recurrent attractors, that are either finite, countable, or a disjoint union of a Cantor set and a countable set. We also show that the attractor of a piecewise contracting map is usually a Lebesgue measure-zero set, and we give conditions ensuring that it is totally disconnected. Finally, we provide an example of piecewise contracting map with positive topological entropy and whose attractor is an interval. This is a joint work with E. Catsigeras, A. Meyroneinc and E. Ugalde.