About the multivariate fractional Brownian motion

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Abstract: Since the pioneering work by Mandelbrot and Van Ness in 1968, the fractional Brownian motion (fBm) has appeared to be a classical stochastic process (used in biology, physics, seismology, hydrology, economics, finance,...) for modelling one-dimensional self-similar or long-memory processes. In particular, we have recently applied this model to characterize the regularity and dependence (through the estimation of the fractal exponent) of fMRI signals acquired in the brain for resting-state patients. This analysis was conducted independently on each region of interest of the brain. Despite the first analysis showed interesting results, the model needed to be considerably improved in order to take into account the possible connectivity of regions of interest. In this talk, we present an extension of the fBm to the multivariate case that may be well-suited to such data: the multivariate fractional Brownian motion (mfBm). The mfBm, denoted by $X(t) = (X_1(t), ..., X_p(t))$ for $p \geq 1$ and $t \geq 0$), is an $\mathbb{R}^p$- valued stochastic continuous process which can be uniquely de fined as follows:

i) $X(t)$ is a Gaussian stochastic process.

ii) $X(t)$ stationary increments.

iii) $X(t)$ is $H = (H_1, ..., H_p)$-self-similar, i.e. $X(ct) = (c^{H_1} X_1(t), \ldots, c^{H_p} X_p(t))$ for any dilation parameter $c > 0$.

After recalling some facts about the fBm, we will state some theoretical properties of the mfBm: (cross)-correlation, spectral density matrix, wavelet analysis, existence conditions. Then, we will detail how we can exactly and quickly generate sample paths of the mfBm. Finally, we will focus on the statistical inference and mainly on the joint estimation of the fractal exponents $(H_1, ..., H_p)$ using a discrete variations techniques. This talks is based on several works in collaboration with S. Achard and P.-O. Amblard (Gipsa-lab, Grenoble University France) and Anne Philippe and F. Lavancier (Laboratory Jean Leray, Nantes University, France).