Asymptotic direction for random walks in mixing random environments

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Abstract: A d-dimensional random walk in a random environment is a Markov chain with state space $\mathbb{Z}^d$ and transition probabilities to nearest neighbors at each site of $\mathbb{Z}^d$ which are random. The collection of transitions at each site of $\mathbb{Z}^d$ is what we call the environment. When the environment has a product law (iid random environment), and the transition probabilities are larger than a constant it is conjectured that

If $\lim X_n \cdot l = \infty$ for some directions $l \in \mathbb{S}^{d-1}$ in an open set,
then there exists a deterministic $v \in \mathbb{R}^d - \{0\}$ such that

$$\lim \frac{X_n}{n} \to v.$$

In this framework an intermediate problem has been solved by Simenhaus, showing that if the first condition of the conjecture holds, then there exists a non zero $\hat{v}$ such that

$$\lim \frac{X_n}{X_n} \to \hat{v},$$

and we call $\hat{v}$ an asymptotic direction. On the other hand, when the environment is not iid but satisfies a mixing condition introduced by F. Comets and O. Zeitouni, in general the ballisticity conjecture is not true and also the proof of Simenhaus breaks down.
In this talk, we will describe sufficient conditions in order to have asymptotic direction for environments satisfying the mixing condition of Comets and Zeitouni.

This is a joint work with Alejandro Ramírez.